

Solutions

QUIZ 8: LESSON 10, 9  
FEBRUARY 6, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Find the general solution to

$$\frac{dy}{dx} + \left(\frac{7}{x}\right)y = 5x + 2.$$

Step 1: Find P, Q

$$P = \frac{7}{x}, \quad Q = 5x + 2$$

Step 2: Find integrating factor

$$u = e^{\int P(x) dx} = e^{\int \frac{7}{x} dx} = e^{7 \ln x} = x^7$$

Step 3: Setup Soln

$$y \cdot u(x) = \int Q(x)u(x) dx$$

$$y \cdot x^7 = \int (5x + 2)x^7 dx$$

$$= \int (5x^8 + 2x^7) dx$$

$$= \frac{5}{9}x^9 + \frac{2}{8}x^8 + C$$

$$\Rightarrow \boxed{y = \frac{5}{9}x^2 + \frac{1}{4}x + Cx^{-7}}$$

2. [5 pts] Find the general solution to

$$t \frac{dy}{dt} - 3y = \frac{1}{t}, \quad t > 0 \Rightarrow \frac{dy}{dt} - \frac{3}{t}y = \frac{1}{t^2}$$

Step 1: Find P, Q

$$P = -\frac{3}{t}, \quad Q = \frac{1}{t^2} = t^{-2}$$

Step 2: Find integrating factor

$$u = e^{\int P(t) dt} = e^{\int -\frac{3}{t} dt} = e^{-3 \ln t} = t^{-3}$$

Step 3: Setup Soln

$$y \cdot u(t) = \int Q(t)u(t) dt$$

$$y \cdot t^{-3} = \int t^{-2} \cdot t^{-3} dt$$

$$\Rightarrow y \cdot t^{-3} = \int t^{-5} dt$$

$$= \frac{1}{-5+1} t^{-5+1} + C$$

$$= -\frac{1}{4} t^{-4} + C$$

$$\Rightarrow y = \frac{-\frac{1}{4} t^{-4} + C}{t^{-3}}$$

$$= -\frac{1}{4} t^{-4} \cdot t^3 + C \cdot t^3$$

$$\Rightarrow \boxed{y = -\frac{1}{4} t^{-1} + Ct^3}$$